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Static Universe in a Modified Brans–Dicke Cosmology

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We derive a static model of the universe, where density and pressure are constant, but the gravitational "constant" and the cosmological term vary with time, by means of the Endo–Fukui modified Brans–Dicke theory.

1. INTRODUCTION

Bertolami (1986) pointed out that the only way to reconcile the values expected by the Glashow-Salam-Weinberg model (Abers and Lee, 1983) for the cosmological "constant" Λ, which is 10^{50} times larger than its present value, or for a GUT theory (Langacker, 1981), where it is expected to be 10^{107} times larger, is to admit a time-varying Λ. Bertolami found a solution for the present universe, and for the radiation phase, given by

$$\Lambda = Ct^{-2} \tag{1}$$

where C is a constant and t stands for cosmic time. Berman (submitted) found this same solution for an equation of state

$$p = \alpha\rho \tag{2}$$

where α is a constant, and p and ρ stand for pressure and rest energy density, respectively. Both cases supposed an energy tensor for a perfect fluid, and the Robertson-Walker isotropic and homogeneous Euclidean case metric. Berman *et al.* (1989) found the same law of variation (1) for a static Euclidean model, pointing out that the study of static solutions was interesting, both for theoretical satisfaction and because if there should be found in the future an explanation for the observed cosmological redshifts other than the expansion of the universe (Peratt, 1990), interest in the static

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cases would gain force. It is very interesting, for instance, to have a constant scale factor, along with constant ρ, while the gravitational constant keeps varying with time (Berman *et al.*, 1989).

We shall now show a model with constant R, ρ, and p where $\dot{G} \neq 0$ and $\dot{\Lambda} \neq 0$. Here, dots stand for time derivatives, and G is the gravitational "constant" from Newtonian theory. It will be seen in what follows that we find again a relation like (1).

Before presenting our model, let us recall that Linde (1974) suggested that Λ should be a function of temperature, as early as 1974.

2. ENDO-FUKUI MODIFIED BRANS-DICKE THEORY

Having in mind a variable Λ, Endo and Fukui (1977) modified the Brans-Dicke Lagrangian so that the following variational principle would apply:

$$0 = \delta \int [\phi(R - 2\Lambda(\phi)) + \frac{16\pi}{c^4} L_m - \frac{w}{\phi} \phi_{,i}\phi^{,i}](-g)^{1/2}\, d^4x \tag{3}$$

where R is the scalar curvature, L_m is the Lagrangian for matter, which is assumed not to depend explicitly on the derivatives of g_{ij}, and ϕ is the scalar field. The authors assumed that

$$\Box\phi = \frac{8\pi}{2w+3} \mu T \tag{4}$$

When $\mu = 1$ we have the original Brans-Dicke theory, and

$$\Lambda = \frac{8\pi(1-\mu)}{4\phi} T \tag{5}$$

goes to zero in that case ($\mu = 1$).

Endo and Fukui derived, for the Robertson-Walker (RW) metric and a perfect fluid, the following relations:

$$\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = \frac{1}{3}\left(\frac{8\pi}{\phi}\rho + \Lambda\right) - \frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} + \frac{w\dot{\phi}^2}{6\phi^2} \tag{6}$$

$$\frac{d}{dt}(\dot{\phi}R^3) = \frac{8\pi}{2w+3}\mu(\rho - 3p)R^3 \tag{7}$$

where, from now on, we shall call R the scale factor of the RW metric, while k is the tricurvature. The gravitational "constant" G is given by

$$G = \frac{1}{2}\left(3 - \frac{2w+1}{2w+3}\mu\right)\phi^{-1} \tag{8}$$

In a particular case, the authors found the same solution given by relation (1) above.

In the next section we shall present a static solution for the cosmological equations.

3. STATIC SOLUTION

When $k=0$ and $\dot{R}=0$, we find the following solution:

$$\rho=\rho_0=\text{const} \tag{9}$$

$$\phi=Bt^2 \qquad (B=\text{const}) \tag{10}$$

$$p=p_0=\text{const} \tag{11}$$

$$\Lambda=Ct^{-2} \tag{1}$$

If we plug this solution into equations (6), (5), and (7), we find that

$$p_0=\frac{\rho_0}{3}+\frac{BC}{6\pi(1-\mu)} \tag{12}$$

$$\rho_0=-\frac{B}{8\pi}(C+2w) \tag{13}$$

In order to satisfy the postulate of energy positivity, and in order to have a positive pressure, we must demand

$$B(C+2w)<0 \tag{14}$$

$$B\left(\frac{C}{1-\mu}-\frac{C}{4}-\frac{w}{2\pi}\right)>0 \tag{15}$$

4. CONCLUSIONS

We found a static model of the universe, with time-varying scalar field and cosmological term, in the Endo–Fukui modified Brans–Dicke theory. This solution differs very little from the one obtained earlier by Berman *et al.* (1989) employing Bertolami's (1986) formalism, where relation (4) may be violated, while the relation between G and ϕ is different from our formula (8) above. This similarity of solutions is not a general property of both formalisms, but is due to the static-R hypothesis, though Berman (1989) also found such a similarity for the exponential inflationary case, when Λ is a constant.

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